An analogy with ideal gas and vacuum

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In this paper, we make an analogy with ideal gas and vacuum. Under this analogy, the inertial mass of the particle comes from its drift mass, and the mass-energy and mass-velocity relations of special relativity are obtained under some hypotheses. Then, we use mass relation to reproduce time dilation, length contraction and Lorentz transformation which are the conclusions of special relativity.

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I. INTRODUCTION

As known to all, Newton described vacuum as an absolute rest space. With the developing of the fluctuation theory of light, people realized that vacuum maybe an elastic medium of light. Unified theory of electromagnetic originated in Maxwell via a special vacuum model in which he believed vacuum had two components, electron ether and magnetic ether. H. A. Lorentz [1] developed the thought of Maxwell's, he separated matter from vacuum, and presented the electron theory which is the top of the combing the electromagnetic theory with ether. But there was a puzzle that all the experiments searching for the evidence of the absolute motion between earth and ether were failed. After the effort of Poincare, Lorentz, and Einstein, a beautiful theory which can explain this puzzle was developed, it names special relativity [2, 3]. Special relativity become a basic theory of modern physics and be applied to many field.

In fact, there are two opinions for Lorentz transformation which is the most important formula of special relativity, one is based on a preferred reference, hold by Poincare, Lorentz, Robertson, Mansouri and Sexl [4–8] *et.al*, another view is that the preferred reference and the absolute time are not exist. The previous view is supported by the discovery of the cosmic background radiation and get ones attention [9]. But how to obtain the same effect in the ether system from classical physics(not from experiments) is an important and interesting problem. This paper we make an analogy with ideal gas and vacuum. Under this analogy, ether can be regard as ideal gas. More to the point, we obtain the energy-mass, velocity-mass relations, the time dilation, length contraction and Lorentz transformation of special relativity from this model under hypotheses, in the same time, we point out that the inertial mass just be added mass in our model. In addition, time dilation and length contraction are important conclusions of special relativity[2, 3] and be used to explain the experiments, such as Michelson-Morley[10] and Kennedy- Thorndike[11]. Length contraction or LorentzCFitzGerald contraction is first posed by Hendrik Lorentz and George Francis FitzGerald as a hypothesis [1]. To understand these phenomena is a very interesting thing.

References [12, 13] propose microscopic structure of black hole which can give some interesting thermodynamical results. In addition, ideal gas analogy with vacuum is very same with the idea of the sound analog of Hawking radiation in fluid [14].

The reference indicate that the space-time inner the black hole has microscopic structure and a natural thought will be the vacuum also has microscopic structure.

In this paper, we make an analogy with ideal gas and vacuum (ideal gas model of vacuum). Under this analogy, we reproduce time dilation and length contraction. In addition, as the most important part of the special relativity, Lorentz transformation is reproduced use our model.

This paper is organized as follows. In Sec. II, we will introduce the ideal gas model of vacuum. Mass velocity relation and mass energy relation will be reproduced in Sec. III. Time dilation and length contraction and Lorentz transformation will be reproduced in Sec. IV and VI respectively. At the last section, we give a discussion.

II. THE IDEAL GAS MODEL OF VACUUM

An ideal gas model of vacuum will be introduced in this section and some interesting results will be given, with hypotheses on the base of the physics of 19th century in the ideal gas model of vacuum as follows:

1. Vacuum can be regarded as an ideal gas.
2. The speed of light is the maximum speed of local in fluid (ideal gas).
3. Particles with mass are spheres which contain the ideal gas, and their states are same with the environment when they rest. The ideal gas inside will be isentropic when moving.

In fact, there are many attempts to explore the relation between fluid and relativity [15–17], electromagnetic theory [18, 19] and achieved some successes. Even the ideal gas model was mentioned in these papers [20] and give some prompts to us.

Special relativity describe some relations between two inertial frame. In order to study these relations. we will give the important equations describing the states of a moving control

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body in ideal gas. Here, we consider one dimension isentropic flow of a control body in ideal gas, it satisfies [21, 22]

$$\frac{T}{T_0} = (\frac{a}{a_0})^2 = (\frac{\rho}{\rho_0})^{\gamma-1} = (\frac{V}{V_0})^{1-\gamma} = (\frac{P}{P_0})^{\frac{\gamma-1}{\gamma}} = 1 - \beta^2, \quad (1)$$

where T is the temperature, P is the pressure, ρ is the density, V is the volume, $\beta^2 = \frac{\gamma-1}{2}v^2/a_0^2 = \frac{v^2}{c_0^2}$, $\mathbf{v}$ is the velocity of the fluid and its size is v, a is the speed of sound. From Eq. 46, we can solve

$$v^2 = \frac{\gamma-1}{2}(a_0^2 - a^2), \quad (2)$$

when $a = 0$,

$$v_{max}^2 = c_0{}^2 = \frac{\gamma-1}{2}a_0^2, \quad (3)$$

so c_0 is the maxim speed of local in fluid, γ is the adiabatic index. The subscript zero correspond to the stagnation state and the moving state without subscript.

Equation (46) presents the relations between the stagnation and moving state for one dimension isentropic flow. One can note that the sonic speed in the rest ideal gas will be slower than its value in the moving one.

III. MASS VELOCITY RELATION AND MASS ENERGY RELATION

When we consider a sphere which has the volume V_0 in the ideal gas accelerated (i.e. $\mathbf{a}$) slowly from static, there will be a resistance on the sphere, which reads [21, 22]

$$\mathbf{F}^* = -m_0^*\mathbf{a}, \quad (4)$$

where $m_0^* = \frac{1}{2}\rho_0 V_0$, which is called added mass or drift mass. and the sphere will satisfies

$$\mathbf{F} = (m_0 + m_0^*)\mathbf{a}, \quad (5)$$

here m_0 is the mass of the sphere. Then we treat the object as a elemental particle, which satisfy the hypothesis 3, and here the ideal gas is vacuum. According to hypothesis 3 and Eq. 2, and considering the motion is an one dimension isentropic flow, we can obtain the equation

$$\frac{\rho_0 V_0 v^2}{2} + \frac{\rho_0 V_0 c^2}{2} = \frac{\rho_0 V_0 c_0^2}{2}, \quad (6)$$

where $\rho_0 V_0 = m_0$. The total energy of the sphere is E_0 which called static enthalpy with the following form

$$E_0 = E_{ks} + E_h = \frac{1}{2}\rho_0 V_0 c_0^2 = m_0^* c_0^2, \quad (7)$$

where E_{ks} and E_h is the kinetic energy and enthalpy of the sphere, respectively. E_0 is an invariant when sphere moves in fluid. In the other hand, the kinetic energy of the fluid (outside of sphere) takes the form [21, 22]

$$E_{kf} = \frac{\rho_0 V_0 v^2}{4} = \frac{m_0^* v^2}{2}. \quad (8)$$

The total energy of the system (contains the total energy of the sphere and the kinetic energy of the fluid) will be

$$E = E_0 + E_{kf} = E_{ks} + E_h + E_{kf} = E_k + E_h, \quad (9)$$

where E_k is the kinetic energy of the system. As E_0 is an invariant, so we can redefine E_{kf} as the kinetic energy of the system

$$E = E_0 + E_{kf} = E_0 + E_k. \quad (10)$$

So the system only presents its added mass to the outside fluid which is considered to be the inertial mass. In this way, the system will satisfies Newton's second law

$$\mathbf{F} = m\mathbf{a}. \quad (11)$$

We deem m the inertial mass in equation (5) which deleted the superscript star here. We must emphasis that equations $m_0^* = \frac{1}{2}\rho_0 V_0$ is obtained in slow motion.

Eq (7) shows that the rest energy which comes from special relativity is the total energy of the object in our model. When a particle moves with velocity $\mathbf{v}$, its volume will dilate and its added mass will become lager according to assumption 3 and equation (5). In order to obtain the relation between added mass and v, we consider a process as follows. In the rest frame(area I in Fig 1), a stationary particle which has the mass M_0 becomes two particles (named B and C) with equal mass $m(v)$(the mass is the function of velocity and denote $m(0) = m_0$) along x axis (see FIG. 1 area I). For the energy is an invariant, so we can obtain

$$E(M_0, 0) = M_0 c_0^2 = 2E(m_0, v), \quad (12)$$

where $E(M_0, 0)$ denotes the total energy of the system and

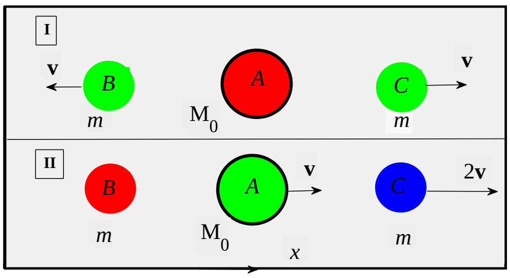

FIG. 1: particle A decomposes into B and C with equal mass m(v). Area I represents the rest frame of A, and Area II represents the mass-center frame of B.

$E(m_0, v)$ is the total energy of the final state particle (B or C), here, the energy is the function of mass and velocity. The kinetic energy of the final state will read

$$E_k = 2E(m_0, v) - 2m_0 c_0^2 = E(M_0, 0) - 2m_0 c_0^2. \quad (13)$$

For the mass is an invariant, ie.

$$M_0 = 2m(v), \tag{14}$$

and the energy reads

$$E(m_0, v) = m(v)c_0^2. \tag{15}$$

Conservation of the mass can be obtained from the following consideration. In the center-mass frame of the particle B(see FIG. 1 area II), we assume that the momentum is still an invariance, so that

$$M_0 v = m(v)(v - v) + m(v)(v + v) = 2m(v)v, \tag{16}$$

here we consider that the mass is an invariant in different frames.

Via the relation between E_k, m, and $m\mathbf{v}$ in one dimension motion, under the force and work defined by $\mathbf{f} = d(m\mathbf{v})/dt$ and $dw = \mathbf{f} \cdot d\mathbf{s}$, respectively,

$$E_k = m(v)c_0^2 - m_0 c_0^2 = \int \mathbf{f} \cdot d\mathbf{s} = \int \mathbf{v} \cdot d(m\mathbf{v}), \tag{17}$$

one can solve that

$$m(v) = m_0(1 - \frac{v^2}{c_0^2})^{-\frac{1}{2}} = m_0(1 - \beta^2)^{-\frac{1}{2}}. \tag{18}$$

IV. TIME DILATION AND LENGTH CONTRACTION

In this section, we will use several thought experiments to deduce time dilation and length contraction.

A. Period motion I: light clock

Firstly, we present a light clock. Just shown in FIG.2, two parallel mirrors are in the $x - y$ plane, and the light reflected between the construct a light clock. The period ratio between the rest clock and the moving one will be

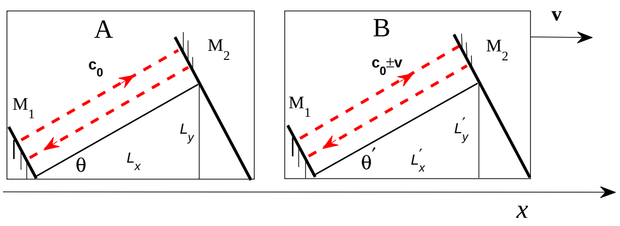

FIG. 2: The rest light clock and the moving light clock are in the left frame(A) and right frame (B) which has a velocity relative to frame(A), respectively.

$$\frac{\tau'}{\tau} = \frac{\sqrt{L_x'^2 + (1-\beta^2)L_y'^2}}{(1-\beta^2)\sqrt{L_x^2 + L_y^2}}, \tag{19}$$

where $\beta^2 = \frac{v^2}{c_0^2}$, v is the speed of the light clock, c_0 is the light speed. τ' and τ are the periods of the rest and moving clock, respectively. L_x, L_y and L_x', L_y' are two components of the rest and moving equipment, respectively. We assume

$$\frac{L_x'}{L_x} = g(v), \frac{L_y'}{L_y} = h(v). \tag{20}$$

x and y direction moving period changes are

$$\frac{\tau_x'}{\tau_x} = b(v) = \frac{g(v)}{1-\beta^2}, \frac{\tau_y'}{\tau_y} = f(v) = \frac{h(v)}{\sqrt{1-\beta^2}}. \tag{21}$$

We assume that the period ratio between a rest and a moving clock just depend on the size of the velocity, i.e.

$$b(v) = f(v) = \frac{h(v)}{\sqrt{1-\beta^2}} = \frac{g(v)}{1-\beta^2}. \tag{22}$$

B. Period motion II: one dimension masses sphere motion

In this section, we consider a moving sphere with speed v_0 in a box with a fully elastic boundary(see FIG. 3) as a clock. The mass of the sphere is $m(v_0) = m_0(1 - \frac{v_0}{c_0}^2)^{-\frac{1}{2}}$ when this clock is rest. If we accelerate the clock to a speed v, $m(\sqrt{v'^2 + v^2}) = m_0(1 - \frac{v'^2+v^2}{c_0^2})^{-\frac{1}{2}}$, and the momentum of the sphere in the direction perpendicular to x axis is a conservation, i.e.

$$m_0(1-\beta^2)^{-\frac{1}{2}} v_0 = m_0(1 - \frac{v^2 + v'^2}{c_0^2})^{-\frac{1}{2}} v'. \tag{23}$$

One can obtain the relation

$$v' = v_0\sqrt{1-\beta^2}. \tag{24}$$

We assume that the length of the equipment M in the sphere moving direction relate to L and L' when M is rest and moving respectively, so the period ratio $\frac{\tau'}{\tau}$ between the rest and the moving clock reads

$$\frac{\tau'}{\tau} = \frac{v_0 L'}{v' L} = \frac{L'}{\sqrt{1-\beta^2}L} = \frac{h(v)}{\sqrt{1-\beta^2}}. \tag{25}$$

Equation (25) just is the special case of the equation (21), and is a test for our previous hypothesise.

C. Period III: circling motion

In this section, we will consider a circling motion as a clock(see FIG.4). Circling motion is in $y - z$ plane. If we assume the rest mass of the moving sphere is m_0, the radius of the circle is r and r' corresponding to case A and case B respectively, obviously, $m = m_0(1 - \frac{\omega r^2}{c_0^2})^{-\frac{1}{2}}$, $m' =$

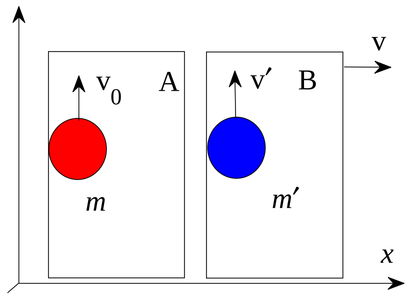

FIG. 3: Period of one dimension moving sphere as a clock(the entire equipment is named M). The rest clock and the moving clock are in the left frame (A) and right frame (B), respectively.

$m_0(1-\frac{\omega' r'^2+v^2}{c_0^2})^{-\frac{1}{2}}$. For there is no force in $y-z$ plane, so the angular momentum is an invariant quantity, one can obtain

$$m\omega r^2 = m'\omega' r'^2, \tag{26}$$

for the period, we can write

$$\frac{2\pi/\omega'}{2\pi/\omega} = \frac{r'}{r\sqrt{(1-\beta^2)}}, \tag{27}$$

$$\frac{\omega'}{\omega} = \sqrt{1-\beta^2}, \frac{r'}{r} = 1, h(v) = 1. \tag{28}$$

At last, the result will be

$$g(v) = \frac{1}{b(v)} = \sqrt{1-\beta^2}, \tag{29}$$

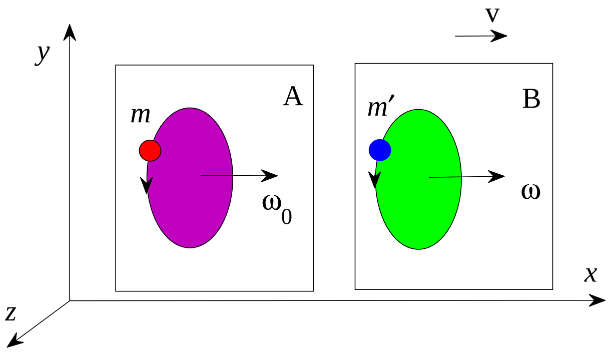

FIG. 4: A circling motion sphere as a clock. The rest clock and the moving clock are in the left frame (A) and right frame (B), respectively.

Combine above three thought experiments, we can conclude following.

1. The length will not change in the direction which the velocity component is zero and will contract $\frac{1}{\sqrt{1-\beta^2}}$ times in the moving direction.
2. The period of the moving clock will dilate $\frac{1}{\sqrt{1-\beta^2}}$ times which is not depends on the direction of the velocity.

V. LORENTZ TRANSFORMATION

In this section, we will divide Lorentz transformation under the previous length contraction and time dilation. At first, we assume that the transformation between different inertial frame showns in Fig. 5, the transformation has the following simple form

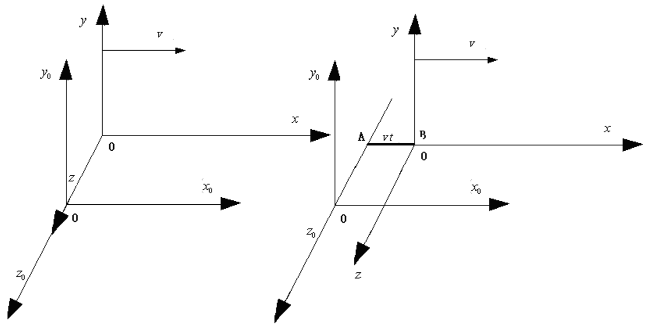

FIG. 5: Different inertial frame, S and S_0 represent moving reference and static reference, respectively, v is the velocity of the S reference, and it s direction along x axes. S and S_0 will coincide when $t = t_0 = 0$.

$$x_0 = a_{11}x + a_{14}t, y_0 = a_{22}y, z_0 = a_{33}z, t_0 = a_{41}x + a_{44}t, \tag{30}$$

yield

$$x = \frac{a_{44}x_0 - a_{14}t_0}{a_{11}a_{44} - a_{14}a_{41}}, y = \frac{1}{a_{22}}y, z_0 = \frac{1}{a_{33}}z, t = \frac{a_{11}t_0 - a_{41}x_0}{a_{11}a_{44} - a_{14}a_{41}}, \tag{31}$$

and the velocity transformation will be

$$v(x_0) = \frac{a_{11}v_x + a_{14}}{a_{41}v_x + a_{44}}, v(y_0) = \frac{v_y}{a_{41}v_x + a_{44}}, v(z_0) = \frac{v_z}{a_{41}v_x + a_{44}}, \tag{32}$$

where the parameter with subscript represent in the static reference. For the length in the vertical of direction of the moving reference, so we have

$$a_{22} = a_{33} = 1. \tag{33}$$

In S_0 frame, S_0 will be zero in $x_0 = vt_0$, one can obtain

$$a_{14} = a_{44}v. \tag{34}$$

In the last paper, our conclusion is that in the moving frame, the length will contract $\sqrt{1-M_0^2}$, but just as our previous assume, the rule will contract same times, so the result of length in the moving frame will not change with it of the same object in the static frame. So the length of the moving object observed in the static will contract $\sqrt{1-M_0^2}$ times than the result of length in the moving frame. We have the relation

$$x_2 - x_1 = \frac{a_{44}(x_{2_0} - x_{1_0})}{a_{11}a_{44} - a_{14}a_{41}} = \frac{(x_{2_0} - x_{1_0})}{\sqrt{1-\beta^2}}, \tag{35}$$

where x_2 and x_1 is the x coordinate of the two end points of the object stay in the S frame, x_{2_0} and x_{1_0} the x coordinate of the two end points of the object stay in the S_0 frame in the same time. and then

$$\frac{a_{44}}{a_{11}a_{44} - a_{14}a_{41}} = \frac{1}{\sqrt{1-\beta^2}}. \tag{36}$$

Similarly, the time of the moving clock observed in the static will dilate $\frac{1}{\sqrt{1-M_0^2}}$ times than the result of time in the moving frame. When the two frame coincide, $t = t_0 = 0$, and in some time, the time of the clock between in $x = 0$ or $x_0 = vt_0$ in S frame and the clock in S_0 frame have the

$$t_0 = a_{44}t = \frac{t}{\sqrt{1-\beta^2}}, \tag{37}$$

then

$$a_{44} = \frac{1}{\sqrt{1-\beta^2}}, \tag{38}$$

for the general case, a clock runs $t_2 - t_1$, and in S_0 frame it will be

$$t_{2_0} - t_{1_0} = a_{44}(t_2 - t_1) = \frac{t_2 - t_1}{\sqrt{1-\beta^2}}, \tag{39}$$

and we obtain the same value of a_{44}.

In order to obtain the last equation, we consider the light frequency and wave length transformation. In the static frame

$$c_0 = \frac{\omega_0}{k_0} = \frac{\omega_0}{\sqrt{k_{x_0}^2 + k_{y_0}^2 + k_{z_0}^2}}, \tag{40}$$

the phase will not change in different frame, which satisfies

$$\mathbf{r} \cdot k - \omega t = \mathbf{r_0} \cdot k_0 - \omega_0 t_0. \tag{41}$$

Submit equation 30 to the last equation, for the arbitrary of x, y, z, t, we have

$$k_x = a_{11}k_{x_0} - a_{41}\omega_0, k_y = a_{22}k_{y_0}, k_z = a_{33}k_{z_0}, \omega = a_{44}\omega_0 - a_{14}k_{x_0}. \tag{42}$$

In the static frame, the light along y_0 axes, the relation will be

$$k_z = k_{z_0} = k_{x_0} = 0, c_0 = \frac{\omega_0}{k_{y_0}}, \tag{43}$$

for the equation

$$c^2 = \frac{\omega^2}{k_x^2 + k_y^2 + k_z^2}, \tag{44}$$

submit equations 42 and 43 to the last equation, one can obtain

$$c^2 = \frac{a_{44}^2}{a_{41}^2 + (\frac{a_{22}}{c_0})^2}, \tag{45}$$

with the help of equation 32, we get

$$c^2 = v_x^2 + v_y^2 = (\frac{a_{12}}{a_{11}})^2 + c_0^2(\frac{a_{11}a_{44} - a_{12}a_{44}}{a_{11}a_{22}})^2, \tag{46}$$

and then, the following equation will be obtained

$$a_{14} = \frac{a_{41}a_{44}a_{11}c_0^2}{a_{22}^2 + a_{41}^2c_0^2}. \tag{47}$$

At last, combine the equations 33, 34, 36, 38 and 47, we have

$$\begin{aligned} a_{11} = a_{44} &= \frac{1}{\sqrt{1-\beta^2}}, \\ a_{22} = a_{33} &= 1, \\ a_{14} &= \frac{v}{\sqrt{1-\beta^2}}, \\ a_{41} &= \frac{\frac{v}{c_0^2}}{\sqrt{1-\beta^2}}. \end{aligned} \tag{48}$$

Finally, we obtain Lorentz transformation

$$x_0 = \frac{x + vt}{\sqrt{1-\beta^2}}, y_0 = y, z_0 = z, t_0 = \frac{t + \frac{xv}{c_0^2}}{\sqrt{1-\beta^2}}. \tag{49}$$

In the same time, the wave vector transformation is

$$k_x = \frac{k_{x_0} - \frac{v\omega_0}{c_0^2}}{\sqrt{1-\beta^2}}, k_y = k_{y_0}, k_z = k_{z_0}, \omega = \frac{\omega_0 - vk_{x_0}}{\sqrt{1-\beta^2}}. \tag{50}$$

VI. CONCLUSION AND DISCUSSION

In this work, we reproduce some important results of special relativity, such as mass velocity and mass energy relations under the simplest ideal gas model of vacuum hypothesis, the inertial mass of the particle comes from its drift mass. In addition, our model has a free parameter γ which is called adiabatic index, and when $\gamma = 3$, the speed of sound in the rest fluid equal to the maxim speed of the fluid ($a_0 = c_0$). In this paper, we also reproduce the time dilation and rule contraction based on mass velocity relation which can be reproduced by ideal gas model, and at last we reproduce Lorentz transformation.

We must emphasis that all the conclusions are obtained under the existing of the ether, i.e. the vacuum is the ideal gas and has a static state. Additional, all the velocity is related to the static vacuum, and the base is classical physics of 19 century. Of cause, this ideal gas model of vacuum is very simple. It cannot describe the complex structure of the elemental particles. Therefor, this model can help us to understand the nature of vacuum, special relativity and the relation ship between them. In addition, it is a explore for the inertial mass origin different from Higgs mechanism obtained from quantum physics.

VII. ACKNOWLEDGMENTS

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